AGENT BASED INPUT-OUTPUT INTERDEPENDENCY MODEL

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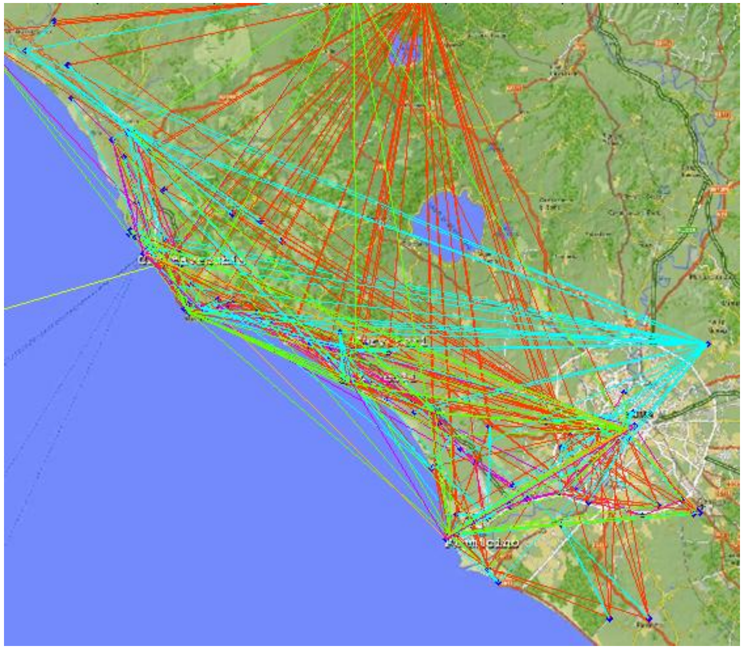
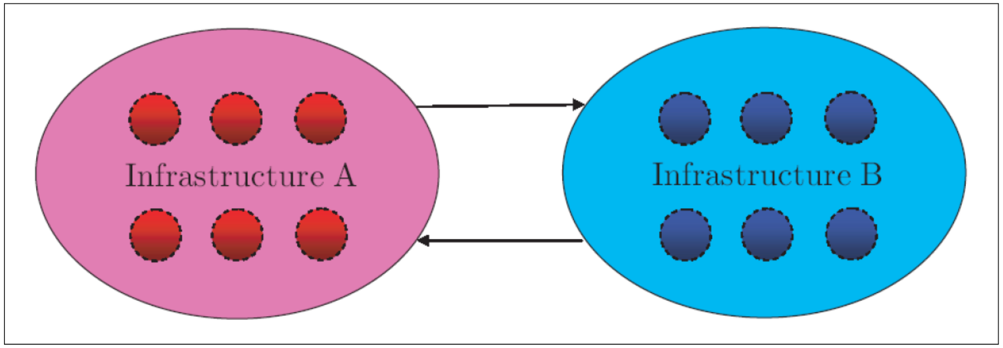
- Conclusions

Interdependecy Modeling

- Represent Mutual Interaction among Infrastructures
  - Detailed sector-specific knowledge
  - Poor understanding of cross-sector interactions
- Typically High-Abstract Models
  - Input-output Inoperability Model
- More detail is required
  - In-the-small models
  - Multi-Layer models

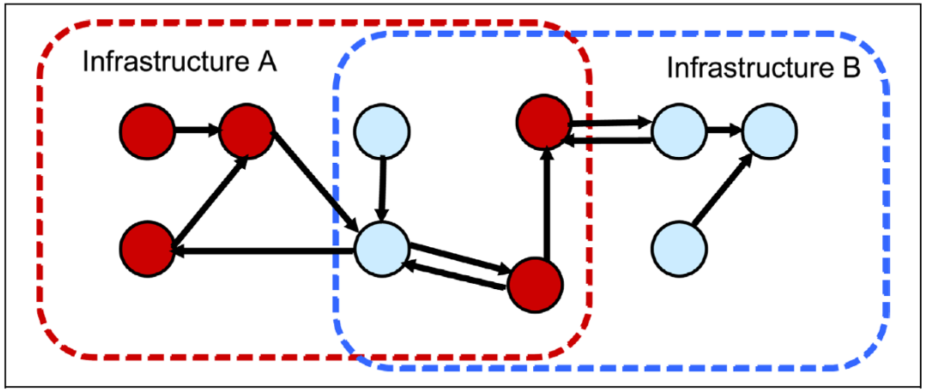


Holistic vs Agent Based Models



Each infrastructure is a reality with its **own identity**, **recognisable boundaries** and functional properties, which interact with other similar entities via identifiable (and reduced) set of relationships.

ABM approaches adopts a **physical perspective** considering the quantities (resources and signals) that are exchanged. The modelled interconnections overlap the frontiers then we **lose the boundary** definitions of each infrastructure



Holistic	Agent Based
High-level	In-the small
Data availability (i.e. BEA)	Lack of quantitative data
Very Abstract	More expressive

Key Ideas

- Agent-based IIM + Resource Exchange = AB-IIM
- The model can be rearranged into a IIM model
- The IIM coefficients are retreived considering:
  - **Resource Requirements**
  - **Resource Production**
  - **Dissipations due to transmission or transportation of resources**

Static IIM

$$\delta\mathbf{x} = A\delta\mathbf{x} + \delta\mathbf{c}$$

$\delta\mathbf{x}$ is the difference between the as-planned $\mathbf{x_0}$ and degraded $\mathbf{x_d}$

Economic Model
(demand-reduction model)

Normalising w.r.t. the as planned production

$$P = [diag\{\mathbf{x_0}\}]^{-1}$$

$$\mathbf{q} = P\delta\mathbf{x}$$

Inoperability Model

$$\mathbf{q} = A^*\mathbf{q} + \mathbf{c}^*$$

The overall inoperability

$$\mathbf{q} = (I - A^*)^{-1}\mathbf{c}^*$$

This solution is feasible only if all $q_i \leq 1$

Dinamic IIM

$$\delta \mathbf{x}(t) = A\delta \mathbf{x}(t) + \delta \mathbf{c}(t) + B\delta \dot{\mathbf{x}}(t)$$

$$B = -K^{-1}$$

$$\delta \dot{\mathbf{x}}(t) = K\left[(A - I)\delta \mathbf{x}(t) + \delta \mathbf{c}(t)\right]$$

K is the *industry resilience* its coefficient be seen as the recovery rate

$$\dot{\mathbf{q}}(t) = K(A^* - I)\mathbf{q}(t) + K\mathbf{c}^*(t)$$

Under which conditions such dynamic system reach an equilibrium ?

Dinamic IIM (2)

A Sufficient Condition for the equilibrium

If $||\mathbf{c}^*(t)||$ is bounded, then a sufficient condition to guarantee the stability of dynamic IIM model is that the maximum of the **dependency indexes** of A^* matrix is less than **one**.

Moreover, if $||\mathbf{c}^*(t)||$ is stationary, the dynamic IIM model reaches an equilibrium this coincides with the static IIM model.

Dependency index

$$\gamma_i = \sum_{j=1}^{n} a^*_{ij}$$

Is the sum on the raw of the Leontief matrix and it represents a measurement of the robustness of the corresponding infrastructure with respect to the inoperability of the others.

Dinamic IIM (3)

Meaning

If the degree of dependency is sufficiently small, then the increment of inoperability is bounded.

The stability of the system does not depend on K. In other terms the *industry resilience* allows to modulate the recovery time but it does not influence the capability of the system to reach an equilibrium (neither the value of such an equilibrium)

Dinamic IIM (3)

Proof

The time evolution of the inoperability can be evaluated as

$$\mathbf{q_f}(t) = e^{K(A^*-I)t}\mathbf{q}(0)$$

$$\mathbf{q_u}(t) = \int_0^t e^{K(A^*-I)(t-\tau)} K\mathbf{c}^*(\tau)d\tau$$

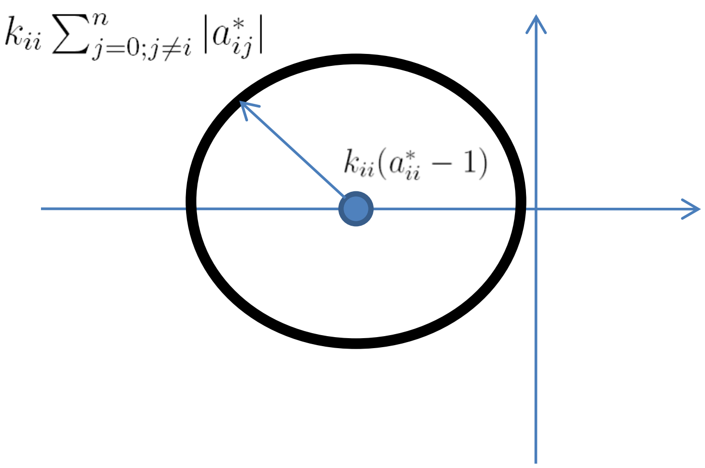
If all the eigenvalues of the matrix $K(A^* - I)$ are no-positive at steady-state the first term vanishes while the second one, when the input is bounded, reach the equilibrium, i.e.

$$0 = K(A^* - I)\mathbf{q_{eq}}(t) + K\mathbf{c}^* \Rightarrow \mathbf{q_{eq}}(t) = (A^* - I)^{-1}\mathbf{c}^*$$

Dinamic IIM (4)

To guarantee that all eigenvalues are no-positive, we use the Circle Theorem that asserts that they are located in a circle centered on the main diagonal element and of radio equal to the sum of the (absolute) value of the row entry, i.e.

$$k_{ii}(a_{ii}^* - 1) + k_{ii} \sum_{j=0;j\neq i}^{n} |a_{ij}^*|$$



By some mathematics it easy to show that the radius can be expressed as

$$k_{ii}(\gamma_i - a_{ii}^*)$$

hence if $\gamma_i < 1$ we are the assert

AB-IIM

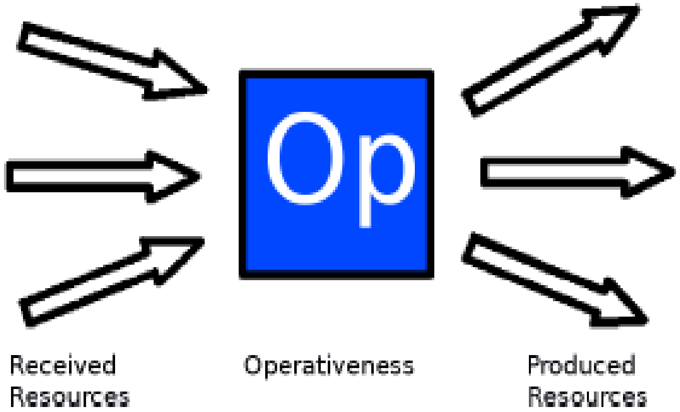
IIM generally refers to infrastructure or sectors as a whole because they are set-up using economic data (exploiting the well documented I/O tables).

Moving to **more detailed models**, e.g. component-wise, the economic relations, are less significant, and no adequate quantitative data can be easily gathered. Indeed validation and set-up are the main drawbacks of Agent-based models

To overcome the limitations of both paradigms, we developed an **Agent-Based IIM Model (AB-IIM).**

In order to obtain a detailed and expressive framework, the **exchange of resources** among the entities is explicitly modeled, while **inoperability becomes an internal parameter.** Nevertheless, the proposed model can be easily transformed into an actual in-the-small IIM model, where coefficients are obtained in a realistic way.

AB-IIM – Operativeness and Resources



Operativeness $\mathbf{op} = \mathbf{1_n} - \mathbf{q}$

IIM in Operativeness Form

$$\mathbf{op} = A^{*}\mathbf{op} + \tilde{\mathbf{c}}^{*}$$

$$\tilde{\mathbf{c}}^{*} = (I - A^{*})\mathbf{1_n} - \mathbf{c}^{*}$$

- ◇ Inoperability becomes an internal parameter
- ◇ Produced Resources depend on operativeness
- ◇ Received resources influence operativeness

Static AB-IIM

n entities and m resource classes

Produced Resources is a (linear) function of the operativeness

$$r_i^j = \phi_i^j op_i \Rightarrow \mathbf{r} = \Phi \mathbf{op} \Rightarrow \mathbf{r} = \Phi \mathbf{1_n} - \Phi \mathbf{q}$$

Operativeness is a (linear) function of the Received Resources

$$op_i = \sum_{j=1}^{m} \psi_i^j \bar{r}_i^j + \tilde{c}_i^* \Rightarrow \mathbf{op} = \Psi \bar{\mathbf{r}} + \tilde{\mathbf{c}}^*$$

Resources can degrade along the path (attenuation/dissipation)

$$\bar{r}_i^j = \sum_{p=1;p\neq i}^{n} \delta_{pi}^j r_p^j \Rightarrow \bar{\mathbf{r}} = \Delta \mathbf{r}$$

Static AB-IIM

AB-IIM Model

$$\begin{bmatrix} \mathbf{q} \\ \mathbf{r} \end{bmatrix} = \begin{bmatrix} 0 & -\Psi\Delta \\ -\Phi & 0 \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \mathbf{r} \end{bmatrix} + \begin{bmatrix} A^* \\ \Phi \end{bmatrix} \mathbf{1_n} + \begin{bmatrix} I_n \\ 0 \end{bmatrix} \mathbf{c}^*$$

It is a *n+m* dimension model

If $\Psi\Delta\Phi = A^*$ the AB-IIM model can be rearranged into a IIM model

$$\mathbf{q} = \Psi\Delta\Phi\mathbf{q} + (A^* - \Psi\Delta\Phi)\mathbf{1_n} + \mathbf{c}^*$$

Dinamic AB-IIM

$$\begin{bmatrix} \mathbf{q}(t) \\ \mathbf{r}(t) \end{bmatrix} = \begin{bmatrix} 0 & -\Psi\Delta \\ -\Phi & 0 \end{bmatrix} \begin{bmatrix} \mathbf{q}(t) \\ \mathbf{r}(t) \end{bmatrix} + \begin{bmatrix} A \\ \Phi \end{bmatrix} \mathbf{1_n} + \begin{bmatrix} I_n \\ 0 \end{bmatrix} \mathbf{c}^* + \begin{bmatrix} B \\ D \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}(t) \\ \dot{\mathbf{r}}(t) \end{bmatrix}$$

$B = -K^{-1}$ $\qquad$ $D = -W^{-1}$

$$\begin{bmatrix} \dot{\mathbf{q}}(t) \\ \dot{\mathbf{r}}(t) \end{bmatrix} = \begin{bmatrix} -K & -K\Psi\Delta \\ -W\Phi & -W \end{bmatrix} \begin{bmatrix} \mathbf{q}(t) \\ \mathbf{r}(t) \end{bmatrix} + \begin{bmatrix} KA \\ W\Phi \end{bmatrix} \mathbf{1_n} + \begin{bmatrix} K \\ 0 \end{bmatrix} \mathbf{c}^*(t)$$

If $\mathbf{\Psi\Delta\Phi} = \mathbf{A}^*$ the AB-IIM model can be rearranged into a standard IIM model. Moreover if $||\mathbf{c}^*(t)||$ is bounded and stationary and if $\gamma_i < 1$ Then the system reaches an equilibrium that coincides with

$$\mathbf{q} = (I - A^*)^{-1}\mathbf{c}^*$$

AB-IIM

The AB-IIM method is a flexible, in-the-small, IIM-based modeling framework;

The parameters tuning is mainly related to the production, transportation and consumption of resources, hence these data can be elicited from stakeholders’ experience

Moreover, it allows to derive the in-the-small Leontief coefficients, transforming the AB-IIM into a simple, although fine grained, IIM model.

In this way one can merge “operative” expertise with economic data to have a double checked validation of the model.

Conclusions and Future Work

AB-IIM model has been introduced to depict the **interdependencies** by decomposing each infrastructure into a set of interconnected elements, and **considering the exchange of resources** among them.

The model is based on **three matrices** that represent the **production**, the **consumption** and the **transmission/transportation** of different resources.

AB-IIM framework, besides providing a detailed model, can be rearranged as a **classical IIM model**, where the Leontief coefficients are derived from operative elements.

Future research will be devoted to the introduction of a ***failure propagation dynamic***, where failures of different origin are spread across the network.

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